Evolution of Conditionally Risk Behaviour

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Abstract. We analysed an adaptive process of conditionally risk behaviour by using an agent-based simulation. We observed that relative high-risk behaviours easily survive in a tournament game. Further, the more high-risk behaviours, but not the highest ones, are most adaptive in a society in which mutants always can invade. These insights not only correspond to human behaviour described by the prospect theory, but also make it possible to explain a rational mechanism which cannot be explained by this theory. While this paper may not be representational, it can provides significant insights beyond behavioural economics to areas as diverse as evolution theory, social psychology, and consumer behavioural theory. Moreover, we propose some ideas that can be used to extend the model to analyse it circumstantially.

1 Introduction

Studies on risk behaviour are one of the most significant topics today. People always face many opportunities to take risks. The financial theory mentions that you must take high risk if you would want to gain a high return. A good example is how people choose between an adjustable rate or not for their mortgage repayments.

Result brings competition in many cases. The winner-takes-all phenomenon in business has increased as a result of the increased competition brought by the Internet [8]. If a strategy of the type of education someone chooses for their children is regarded as a kind of risk, schools or firms they enter or work are the results of competition. School entrance examinations and job taking can be regarded as a 'winner takes all' game. In such a competition, the decision on who wins depends on just that whether a player's payoff is larger or smaller than that of their opponent. Here, we discuss a radical type of competition such as a situation where the player's risk behaviour divides the winners from the losers.

We will provide a framework that consists of an evolutionary adaptive process of risk behaviour on the basis of the competitive aspect of the results of games. Our study may be highly nonrepresentational. However, discussions about special traits of how humans manage risk behaviours from the viewpoint of evolution provides an effective theoretical insights with which researchers can rationally describe risk behaviour.

Studies on risk behaviour are often empirical. [9] empirically tested the existence of risk behaviour. Many types of risk studies treat humans' psychological traits by using experimental or questionnaire approaches. This is because the risk behaviour often seems irrational. As well known, the humans often behave irrationally. The prospect theory mentioned how people make decisions among alternatives that involve taking risks such as decision-making in finance [5]. However, the theory has limited explanatory power because it does not explain why the facts emerge. [12] discovered that the free products (price zero products) have a special effect. However, they cannot explain the mechanism of why the free products are special.

Are there any studies in which the reasons for risk behaviour are discussed? Some economic research addresses this question: [10] uses mathematical analysis to explain that people's traits of small risk-averse attitude are a sufficient condition for those of large risk-averse attitude if their utility functions are convex and increasing. [2] extended the expected utility theory in economics. They analyzed how having one's aspirations satisfied affects how consumers perform. These previous works described the empirical facts but did not explain the mechanisms. That is to say, why do these facts emerge? We analysed the risk behaviour on the basis of the adaptive process. What type of risk behaviour brings victory? If this behaviour would be hard for people to calculate rationally, it is important to behave adaptively. Therefore, we will understand that the irrational actions the people do are, in fact, rational in the long term [3] if we analyse the adaptive risk behaviour.

Our work is strongly inspired by [11]. They indicate the advantage of using specific strategies in a tournament game. However, this insight is too restricted to be applicable to more general risk behaviour. To address the goal of evolutionally analysis of the risk behaviour, their work needs to be extended in several ways.

2 Basic Model

Our work deals with evolutionary adaptive process of conditionally risk behaviour as an example of risk behaviour. By extending our idea, our model can be applied to various types of risk behaviours. First, we defined a risk game and a sequential risk game, and constructed a basic model. The adaptive process of the game is then described, and finally our basic model is analyzed and interpreted.

2.1 Definition

First, we define an α-game to develop our model. An α-game is defined as a lottery (gamble or prospect) which either gains or loses α dollar(s) with equally probability, .5. Thus, the α-game is a game the expectation of which is 0 dollars, and the degree of α means that of risk.(α must be a non-negative.) However, we treat only 0-game and 1-game here for simplicity. Second, we use an expectation vector of a lottery as the form $v = (x_1, p_1; \cdots; x_n, p_n)$, denoting an option that gains income x_i with probability p_i. Moreover, the incomes are ranked from least to best in terms of preference order, $x_1 < x_2 < \cdots < x_n$. For example, the form of the α-game is $v = (-\alpha, .5; \alpha, .5)$.

Next, we define n-sequential game to contextualize this game to conditionally risk behaviour. The term "sequential" means that a player can choose the next action after knowing the result of the previous game(s). This idea is used to model conditionally risk behaviour.

As shown in Table 1, when $n = 2$, there are six kinds of game. All the players can choose any game they like. Their selection is called a strategy.

Table 1: All strategies and their expectation vectors in $n = 2$

No.	α of first game	α of sequential game	Expectation vector
I	0	0	$(0, 1)$
II	0	1	$(-1, .5; 1, .5)$
III	1	0	$(-1, .5; 1, .5)$
IV	1	1	$(-2, .25; 0, .5; 2, .25)$
V	1	0 if win in first game, 1 otherwise	$(-2, .25; 0, .25; 1, .5)$
VI	1	1 if win in first game, 0 otherwise	$(-1, .5; 0, .25; 2, .25)$

2.2 Adaptive process

There are many models of the learning process in evolution theory. Here, we use a genetic algorithm (GA) for the learning process. This is because our objective, finding the best conditionally risk behaviour is difficult to discuss calculatively because of the limits to human rationality. To do this, we need a discussion on the evolutional perspective. Therefore, GA as an evolutionary metaphor [4] is appropriate for such an ecological perspective. Further, GA has a lot of variations and we can use it to interpret many situations [6].

Deciding on the selection pressure is very important for adopting GA. According to [6], the models used to decide it are a roulette selection, an expectation selection, a rank selection, an elite keeping selection [1], and a tournament selection. The roulette selection, which is a basic model of decision of selection pressure, is a selection rule, which is used to calculate the survival probability of an individual in next generation in proportion as its fitness value. The expectation selection is usually used when the number of individuals is relatively low, so it is not appropriate for our purpose. This is because the expected value of any *alpha*-game is equally zero. The rank selection is an essentially similar rule to the roulette selection and the elite keeping selection is one of the extensions of the roulette selection. The tournament selection is a rule that allows the survival only the best individual of groups and be copied from the best into the others.

Finally, we have to discuss mutation in GA. It is natural to think that the object in this paper is in an environment marked by natural selection, that is, we must develop our model in which invasion by the new strategists potentially happens. Thus, the mutation process in GA is used here.

2.3 Analysis and Implementation

Before using the evolutional process, we analyzed six strategies defined in Section 2.1 in the tournament game. First, we define the notation $>$ and $=$. Let strategy x and y be ones of the above six strategies. Let $x > y$ hold if and only if the probability x triumphs over y in a 2-players tournament game exceeds .5. Note that $>$ does not satisfy transitivity. Also, $x = y$ if and only if it is just .5. The following relational expressions are then formed between the above six strategies;

$$V > (I, VI) > VI \quad \textit{and} \quad x = y \quad \text{in all the other combinations } (x, y). \qquad (1)$$

Thus, in the tournament game of conditionally risk behaviour, a specific strategy is advantageous to the other strategies while all the strategies are indifference in the roulette game. This result predicates that there are strategies that have higher probability to win the others in a tournament game among the strategies that can set the degree of risk sequentially in spite of having the same expected value. It is very important to discuss this idea to extend the analysis evolutionally.

Are there such rules in our society? A soccer game divides teams into a winner and a loser regardless of hairbreadth margin or double score. A ranking system is used to line up people and even items ranging from cars to corn. An admission exam discriminates successful applicants from rejected ones. Production goods are either bought or put away to the stock. We can call this scenario a tournament game, and strategies in the conditionally risk behaviour differ in terms of the outcome they confer on the players. More noteworthy is the point that the human's simple rationality cannot calculate which strategy is advantageous. However, the tournament games remain ingrained in our society over the course of recorded human history because humans have been participating in the tournament games to get a partner. Therefore, we expect that the humans may prefer advantageous strategies adaptively when they play the tournament games.

3 Simulation of Many-sequential Model

To discover the more detailed advantageously of an adaptive strategy in a conditionally risk behaviour game, we extended the basic model from a 2-sequential game to an n-sequential one. First, we formulate our idea to GA. A player's strategy is defined as a gene of an individual. Theoretically, gene length is $2^n - 1$ and the meaning of every gene is as follows:

Gene = (1, 2(w), 2(l), 3(ww), 3(wl), 3(lw), 3(ll), ...) .

For example, notation [3(wl)] means the [third] selection if the player won [w] the first game and lost [l] the second game. As mentioned in Section 2, alternatives the players can choose are only two types: 0-game or 1-game. The number of all the strategies, i.e., all the phenotypes, is $2^{2^n - 1}$. Thus, when $n = 2$ (2-sequential model), $(\#G, \#T) = (3, 8)$, and when $n = 3$, $(\#G, \#T) = (7, 128)$, and when $n = 4$, $(\#G, \#T) = (15, 32768)$. Strictly, 0-game has no victory or defeat and, therefore we define that a player always wins when they play a 0-game. This is why a gene (000) and (001) denote the same strategies (correctly, (001) is meaningless).

3.1 Observation

We simulate our extended model. All the simulations run 10,000 times with different random seeds and the observation data are their averages. First, we set the parameters as follows: Population is 2000, No. of Generation is 1000, and Mutation rate (which is only used in Section 3.2) is 0.01.

We observed the population ratio of the strategies with a time series. First, some basic strategies were observed in a 2-sequential game. The performances of six strategies in Section 2.1 are shown in Figure 1. As in Eq. 1, Strategy V is the most advantageous and VI is the least, and II and III remain because these are indifferent to all the others. Second, we compared the performances in $n = 2$ (2-sequential game) with those in $n = 3$. Those of the same six strategies in Section 2.1 are shown in Figure 2, I (Always-safe, (0000000)), IV (Always-risky, (1111111)), II (0001000), III (0100000), V (0100100), and VI (0101000). In a 3-sequential game, there were 128 types of strategies; therefore, even Strategy V was not the best and it disappears in due course.

Who won? We observed some notable strategies as shown in Table 2.

Table 2: Strategy name in $n = 3$

Name	Gene type	Expectation vector
Strategy A1	(1010001)	$(-3, .125; -1, .125; 0, .25; 1, .5)$
Strategy B1	(1010011)	$(-3, .125; -1, .25; 1, .625)$
Strategy C1	(1011001)	$(-3, .125; -1, .125; 0, .5; 2, .25)$
Strategy D1	(1011011)	$(-3, .125; -1, .25; 0, .25; 1, .125; 2, .25)$
Strategy C2	(1110001)	$(-3, .125; -1, .125; 0, .5; 2, .25)$
Strategy D2	(1110011)	$(-3, .125; -1, .25; 0, .25; 1, .125; 2, .25)$
Strategy D3	(1110101)	$(-3, .125; -1, .25; 0, .25; 1, .125; 2, .25)$
Strategy E1	(1110110)	$(-2, .25; -1, .25; 1, .25; 2, .25)$
Strategy B2	(1110111)	$(-3, .125; -1, .375; 1, .25; 2, .25)$

As shown in Figure 3, two strategies (B2 and B1) maintain an advantage during relative early periods ($100 \leq t \leq 200$); in particular, Strategy B2 grows and accounts for half of all the populations. However, these Strategy B1 and B2 lost their power and finally arrived at the second best position. In the last period of the simulation, Strategies D1, D2, and D3 have the top share.

3.2 Mutation

As mentioned in Section 2.2, we need to test an environment with a mutation. Here for simplicity, mutation emerges in a level of an individual, not a level of a gene. Performances of the installed mutation (Figure 3) are shown in Figure 4. Compared with performances without mutation (Figure 3), the strategies which were advantageous in the initial stages maintained their advantage until the end of the simulation. However, the strategies that were of advantage at the end of the simulation could not rise to the top of the share.

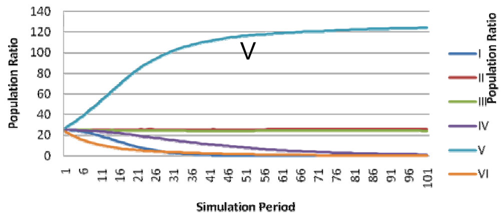

Figure 1: Population ratio in every strategy in 2-sequential game

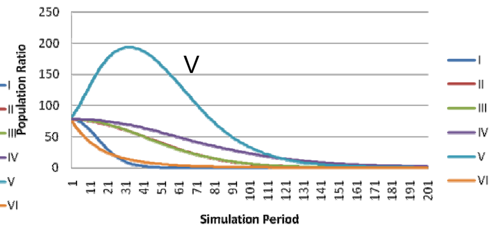

Figure 2: Population ratio in 3-sequential game (1)

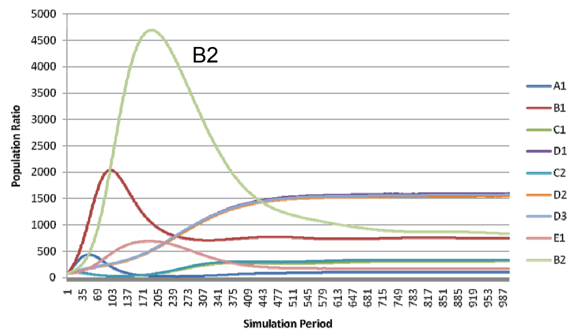

Figure 3: Population ratio in 3-sequential game (2)

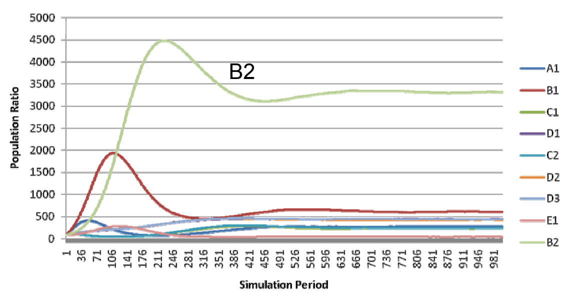

Figure 4: Population ratio in 3-sequential game with mutation

3.3 Discussion

We now discuss the simulation results of our extended model. First, we clarified that relative high-risk strategies are the most suitable in the tournament game. However, the highest risk strategy (1111111) disappeared during the early periods, so the best degree of risk is in intermediary position. Moreover, under the circumstance with mutation, the best strategies became more risky. These are very interesting and significant insights, we need to analyse them in more detailed.

Second, we consider why the mechanism emerges. In the society without mutation, Strategy B1 extends its influence rapidly in the early periods. This is because the probability of that the expected value exceeding zero is .625, and then it triumphs over the other strategies accounting for over half of all. As a result, Strategy B1 has a 20 % share of the pie at $t = 100$. After $t = 100$, a lot of strategies are dumped into the dustbin. What comes next? According to the notation in Section 2.3, the following relational expressions are formed:

$$Dx > B2 > B1 \quad and \quad B1 = Dx \quad (x = 1, 2, 3). \tag{2}$$

Strategy B2 has a unique and peculiar trait, which is indifferent or dominant to almost all the other strategies and triumphs over B1, that is, it has a property similar to the Evolutionally Stable Strategy [7]. Therefore, B2 has a 47 % share of the pie at $t = 200$.

After Strategy B2 came to the top, the model was the most interesting. As shown Eq. 2, Strategy Dx can kill B2 in the society without mutation. This resembles the way that mammals triumphed over dinosaurs in evolutionary history even though they were not the most powerful phylon. This phenomenon cannot be seen in the society with mutation. This is because in our society, a certain number of mutants is always invading, and so using Dx cannot lead to a victory over them.

We will now discuss why a strategy to act conservatively after taking a risk and wining the first stages, did not result in some benefit. Studying the performance of Strategy A1, a representative example of this strategy, provides us with a hint. As is clear from Figure 3, the share is expanded in the early simulation periods temporally, and is kept, albeit only slightly, in the final period. However, according to Eq. 3, A1 cannot triumph over B1 and B2, which are dominant in the early periods. It does not become extinct because it triumphs over Dx.

$$(B1, B2) > A1 > Dx \quad (x = 1, 2, 3) \tag{3}$$

Such a sensitive insight into risk behaviour provides an effective discussion about not only behavioural economics but also evolution theory, social psychology, and consumer behavioural theory. The prospect theory mentioned that people tend to act safe when they gain high payoffs and vice versa. Our study clarified its mechanism that cannot be discussed in the context of the prospect theory.

4 Ideas for Extensions

We need the more extended model, and now present some ideas for extensions. First, a model can be used to treat many types of α-game will be considered. For example, a four-type model ($\alpha = 0, 1/3, 2/3, 1$), a gene of player's strategy is formulated as

Gene = (1(2bits),2(w)(2bits),2(l)(2bits), ...).

The gene length is then six bits and the number of its phenotype is 64. According to our preparatory experiment, Strategy $(1, 1/3, 1)$, $(1, 0, 1)$, and $(2/3, 2/3, 1)$

finally won in the four-type model. Moreover, Strategy $(1, 3/7, 1)$ remains in competition in the eight-type model. Another extensionis continualising the degree of the risk. As you might be aware, the expectation vectors of Strategy Dx (D1, D2, and D3) are all the same. Therefore, we have received approval to investigate which vector is the most suitable expectation vector. The work described in this paper will be extended by searching for an evolutionally stable strategy. We will analyse a one-shot risk game, which follows a probability density function that has a payoff with an expected value of zero. Whether a conditionally risk behaviour which fulfils the expectation vectors exists or not is considered to be a separate issue. We need to decode the inverse problem. By analysing such extensions carefully, we can contribute to a significant discussions about the most suitable conditionally risk behaviour.

Acknowledgement

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